

Low Key

What is a zero-multiversal number? Here's an ever useful first step.

Standard Approach: Brute Forcer

Trying to understand a weird property? Brute force all (small) numbers with that property—concrete data makes pattern matching easier!

If you try counting or printing out all the zero-multiversal numbers less than 10^6 ... you'll find that it's *most* of them, actually. Python is excellent for churning out standard code concisely.

```
from itertools import permutations
n = 10**6
zero_multiversal = []
for v in range(1, n):
    if any(
        '0' in str(abs(int(''.join(p)) - int(''.join(q))))
        for p in permutations(str(v))
        for q in permutations(str(v))
        if p != q and p[0] != '0' and q[0] != '0'
    ):
        zero_multiversal.append(v)

print(len(zero_multiversal))
for v in zero_multiversal:
    print(v)
```

Try running this code yourself. You'll get a huge amount of output!

```
999810
102
103
104
105
106
107
108
109
112
113
... (it looks like almost all integers are here...)
999995
999996
999997
999998
```

Isn't that suspicious? Modify the brute forcer to find all **non-zero-multiversal** numbers $< 10^6$ instead, and see what you get.

Please, actually do it and try running it. It's something you have to do and see for yourself.

We see that there seem to be four kinds of non-zero-multiversal number:

- < 100
- $dd0$ or $d0d$ (three-digits only)
- $d000\dots00$
- $dddd\dots dd$

If you were solving purely with pen and paper, it might take a while to realize the third and fourth bullets... and it might be *extremely tricky* to realize the annoying edge case that is the second bullet. But with a brute forcer program, the pattern is obvious!

The first two bullets are known to be true because our computer bashed them. We make this conjecture for the “larger” non-zero-multiversal numbers:

Claim

A positive integer with four or more digits is **not** zero-multiversal *if and only if* it looks like:

- $d000\dots00$ or
- $dddd\dots dd$

for some digit d .

The proof is deferred until later (in a short contest, you might even feel like risking it and submitting without proof). The important fact is that there are very few non-zero-multiversal numbers $\leq n$ (around $O(\log_{10}(n))$, because it has to do with digits).

So, we have this simple logarithmic time solution. First, generate all non-zero-multiversal numbers. Then, for each test case:

- Initialize `ans := r - 1 + 1` (assume all in range are zero-multiversal)
- Iterate over all non-zero-multiversal numbers, and do `ans -= 1` for each non-zero-multiversal number in $[\ell, r]$.

The main idea of the proof (of the claim earlier) is that $d - d = 0$ for any digit d , so all we have to do is ensure that our constructed 0 digit is not a leading zero, and that our two constructions are different numbers.

Proof: (of the claim earlier)

Numbers of that form are not zero-multiversal because you can't permute their digits to get another different number.

On the other hand, suppose a number is not of that form. Then, we can always find three digits a and b and c such that

- at least two of them are nonzero, and
- the three are not all equal to each other,

and we can also select any other fourth digit d .

WLOG suppose a and b are the nonzero digits, with $c \neq b$. Then, constructively, $\mathbf{p} := \mathbf{abcd} \dots$ and $\mathbf{q} := \mathbf{acbd} \dots$ works, with 0 as the fourth digit (from the left) in $|p - q|$ (and is not a leading zero).