Glacier Adventure

Problem Solving Technique: Play

Play around with actual concrete cases on pen and paper. See if there are any patterns you can notice among them.

Problem Solving Technique: What does it do?

Have a strange operation? Try rephrasing it in more concrete terms:

- What is its effect on the other things in the scenario?
- What does it enable you to do, in terms of these other things?

It may also be helpful to describe what it *doesn't* do.

Hopefully, you should smell something fishy after playing around with it.

Claim

It's always better (or at least, never bad) to end on a Balance operation.

The basis of this claim comes from the following fact:

Claim

Given a fix sum s, among all pairs of integers x, y such that $x + y = s$, the product xy is maximized when $x \approx s/2$ and $y \approx s/2$.

A formal proof of this is deferred until later, but it should be intuitive geometrically: a square maximizes the area among all rectangles of a fixed perimeter (see the visuals of the sample image!).

Here's the kicker. If we're going to end on a Balance operation, then our sole goal is to maximize the sum $x + y$ of the coordinates as much as possible.

- Balance and Climb and Dive are useless (other than the final Balance at the end).
- For the action cards, sort them by $x_i + y_i$ and just take the p ones with the largest such sums.

Proof: (of the max product claim)

WLOG suppose $x \leq y$, and suppose $y - x > 1$. Then, we can improve the product by bringing them closer to each other. We can show that

$$(x+1)(y-1) > xy$$

by starting at

$$y - x > 1,$$

adding xy to both sides, and rearranging.

Therefore, the product is maximized when $y - x \leq 1$, which happens when (if $x + y = s$)

$$x = \left\lfloor \frac{s}{2} \right\rfloor,$$
$$y = \left\lceil \frac{s}{2} \right\rceil.$$