A Penchick's Tale

Problem Solving Technique: Why is this hard?

Explore straightforward "just do it" approaches or thought processes, even if they seem wrong. Articulate precisely *what's stopping you*, i.e. what is the heart of what makes the problem hard—this gives you something concrete to try to attack.

Can't I just make the maximum amount of each of squawk and chirp and quack, as much as possible? Making more is always better, right?

Unfortunately, as stated, this solution is ill-defined, because quack conflicts with the letters in squawk and in chirp.

So, what makes the problem hard?

- The number of quack we construct affects the number of squawk and chirp we can construct, which makes things messy.
- The problem is hard because there is a tradeoff made whenever we make a quack instead of a squawk and chirp.
- We can't just always max out on one option—we need to find "the right number" of quack to make.
- How to compute this magic number is not obvious (and depends on the values of a and b and c)—it feels hard to express in a formula.

But while thinking about conflicts, we might notice that...

- If I'm only considering squawk and chirp, their letters don't conflict.
- So, considering just them, I can greedily make as much of each of them, since they don't interfere with each other.

The problem has now been reduced to this key goal:

• Find the correct number of quack to make.

We have a standard tool for that.

Standard Toolbox: Complete Search

Can't figure out the right value? Just try everything.

- **Pros:** Always works
- Cons: Might be too slow (do the analysis)

We now have the following solution sketch.

For each z from 0 to (max no. of quack that can be made):

- Make z copies of quack
- Using the remaining tiles, makes as many squawk and chirp as possible; let these values be x and y.
- The maximum happiness in this case is ax + by + cz.

The answer to the problem is the maximum value of ax + by + cz across all possible values of z.

The maximum no. of quack that can be made is definitely $\langle |s|$. Each iteration of the loop can be done in O(1).

Thus, this solution runs in O(|s|), which gets 100 points.

Remark

There are many other greedy solutions to this problem, but it's trickier to prove their correctness.