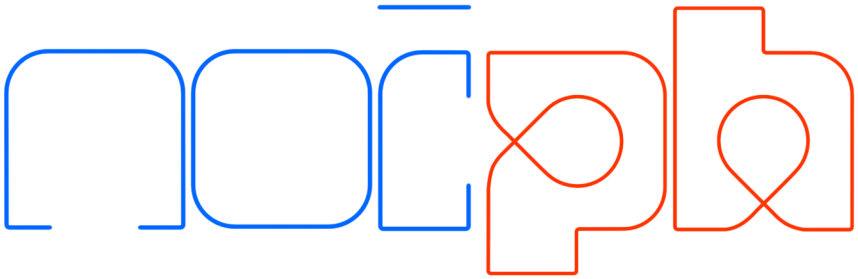


2024

National Olympiad in Informatics

TAMa

TAMA

Important! Read the following:

- Unlike in other NOI.PH contests, your answer to each subtask is a single number, **not code.**
  - For each subtask, substitute in that subtask's value of n (or m, or whatever parameters) and solve the problem, given that value.
  - Type **only the final answer** into the input field for that subtask.
  - When making a submission, you may choose to answer only one subtask at a time (leaving the other fields blank). Your total score counts a subtask as solved if *any* previous submissions have solved that subtask.
- **Do not** put commas or spaces in numerical output. For example, if the problem is to compute 1111×1111, submit `1234321` **not** `1,234,321`.
- It is **highly recommended** to aim for 100 points in every problem before going for 150 or 200 points in most problems.
- Each problem has been designed according to a "one-minute rule".
  - We guarantee that for all subtasks of all problems, a sufficiently efficient algorithm exists that allows the answer to be obtained on a modestly-powered computer in **less than one minute**.
  - Of course, for particularly tricky problems, it may take you much longer than that to *come up* with the solution!
  - Although this rule will not be strictly enforced, we greatly encourage you to try and follow it! You'll learn new and interesting techniques if you choose to take this path, and we believe it is the most fun!
- You are encouraged to use the internet as a resource while solving these problems. We hope that in your research for this contest, you come across and learn many new topics in mathematics!
- The perfect score for this contest is at least 2000 points... possibly more!
- Finding the problems too hard? Please join our upcoming **Abakoda 2024 Beginner Programming Contests** which start in November! Stay tuned in our Discord for more details.
- Good luck and enjoy the contest! 😃

Contents

Problem A
Nostalgia

Nostalgia is a finicky thing. There's a certain sweetspot where a memory has to be old enough that you can look fondly back on it with rose-tinted glasses, but not *so* old that too much of the details and strong emotions start to fade away.

That said, given some integer k, let $r_1, r_2, \ldots, r_{2k-1}$ be the *rose-tinted sequence* of length $2k-1$, where

$$r = [1, 2, 3, \ldots, k-1, k, k-1, \ldots, 3, 2, 1].$$

Then, let $m_1, m_2, m_3, \ldots$ be the infinitely-long *memory sequence*, where

- If $1 \leq n \leq 2k-1$, then $m_n = n$.
- Else, if $n > 2k-1$, then $m_n = r_1 m_{n-1} + r_2 m_{n-2} + \cdots + r_{2k-1} m_{n-(2k-1)}$

For example, if $k = 3$, then the first $2 \cdot 3 - 1 = 5$ terms of m are $[1, 2, 3, 4, 5]$. Then,

$$\begin{aligned} m_6 &= r_1 m_5 + r_2 m_4 + r_3 m_3 + r_4 m_2 + r_5 m_1 \\ &= 1(5) + 2(4) + 3(3) + 2(2) + 1(1) \\ &= 27. \end{aligned}$$

You can also then check that $m_7 = 57$.

Please compute the sum $m_1 + m_2 + \cdots + m_N$. This number can get quite huge, so just output the remainder when this result is divided by the value 998244353.

You can check that when $N = 7$ and $k = 3$, the answer is $1+2+3+4+5+27+57 = 99$.

Subtasks

To be awarded the points for each subtask, answer the problem, given that subtask's values of N and k.

Subtask	Points	Constraints
1	**50**	$N = 10$ and $k = 3$
2	**50**	$N = 10^5$ and $k = 42$
3	**50**	$N = 10^8$ and $k = 101$
4	**50**	$N = 10^{18}$ and $k = 123$

Problem B
Holy Molé

Lóla Lōla wants to make the best karé-karé in the Philippines. So, she lays out 2^n bowls of karé-karé in a row and prays to the heavens.

Then, n angels appear, whom we've labeled 0 to $n-1$.

- The first angel touches the first bowl, then skips the next bowl, then touches the next bowl, then skips the next bowl...
- The second angel touches the first two bowls, then skips the next two bowls, then touches the next two bowls, then skips the next two bowls...
- The third angle touches the first four bowls, then skips the next four bowls, then touches the next four bowls, then skips the next four bowls...
- And so on...

In general, angel i alternates between touching 2^i bowls in a row and skipping 2^i bowls in a row, departing when there are no more bowls left to touch.

For example, if $n = 3$, you can verify that:

- The first bowl is touched 3 times
- The second, third, and fifth bowls are each touched 2 times
- The fourth, sixth, and seventh bowls are each touched 1 time
- The eighth bowl is never touched.

How many bowls have been touched by at least m different angels? This number can get quite huge, so just output the remainder when this result is divided by the value 10000019.

You can check that when $n = 3$ and $m = 2$, the answer is 4.

Subtasks

To be awarded the points for each subtask, answer the problem, given that subtask's values of n and m.

Subtask	Points	Constraints
1	**50**	$n = 10$ and $m = 7$
2	**50**	$n = 60$ and $m = 50$
3	**50**	$n = 10^7$ and $m = 6666666$
4	**50**	$n = 202400769120730662$ and $m = 101200384561365331$

Problem C
The Amazons in their Prime

Maxi and Mini are twin sisters who are recreational mathematicians, but primarily they are warriors who defend their tribe. In order to keep their bodies in peak condition, they make sure to regularly exercise in between their math studies.

Suppose that in some exercise session, Maxi and Mini perform some *set* that is intended to last for n seconds. When Maxi and Mini exercise together, they start at the same time, and attempt to do as many *reps* as possible (with no break between each rep), where each rep takes k seconds to complete. But suppose that the timer for n seconds runs out while they are in the middle of a rep.

- Maxi toughs it out and will spend extra time to complete this last rep.
- Mini gives up early and won't finish this last rep.

For example, suppose $n = 50$ and $k = 7$. Maxi and Mini will have finished seven full reps, but 1 second into the eight rep, the timer runs out. Then, Maxi will finish and complete the eighth rep, but Mini will not (so she has completed seven full reps only). As you can imagine, over time, the total number of reps completed by Maxi will outpace the total number of reps completed by Mini.

Suppose that Maxi and Mini have a number of exercise sessions together, one for each pair of integers (n, k) such that:

- $\ell_n \le n \le r_n$ (that is: the set length, in seconds, is in $[\ell_n, r_n]$)
- $\ell_k \le k \le r_k$ (that is: the rep length, in seconds, is in $[\ell_k, r_k]$)

In total, this means Maxi and Mini do $(r_n - \ell_n + 1) \times (r_k - \ell_k + 1)$ exercise sessions together. Across all these sessions, let M be the total number of reps completed by Maxi, and let m be the total number of reps completed by Mini. What is $M - m$? This number can get quite huge, so just output the remainder when this result is divided by the value 998244353.

For example, when $\ell_n = 15$ and $r_n = 20$ and $\ell_k = 6$ and $r_k = 12$, you can check that $M = 104$ and $m = 66$, so the answer is $M - m = 38$.

Subtasks

To be awarded the points for each subtask, answer the problem, given that subtask's values of ℓ_n and r_n and ℓ_k and r_k.

Subtask	Points	Constraints
1	**50**	$\ell_n = 98$ and $r_n = 100$ and $\ell_k = 8$ and $r_k = 10$
2	**50**	$\ell_n = 1008$ and $r_n = 2024$ and $\ell_k = 6$ and $r_k = 1001$
3	**50**	$\ell_n = 5 \times 10^7$ and $r_n = 10^8$ and $\ell_k = 2$ and $r_k = 5 \times 10^7$
4	**50**	$\ell_n = 5 \times 10^{14}$ and $r_n = 10^{15}$ and $\ell_k = 2$ and $r_k = 5 \times 10^{14}$

Problem D
SRS

Alice prepared a comprehensive list of "standard" topics that she expects everyone in her math and competitive programming club to master by the end of the year (*if they want to take themselves seriously*, of course). Bob took one look at the length and scope of this list, and knew that Alice was *way too ambitious*.

He texted her, "`Alice, r u SRS???`"

Alice replied, "Yes! The *Self-Referential Sequence* is totally standard!"

The SRS (Self-Referential Sequence) $s_1, s_2, s_3, \ldots$ is the positive integer sequence that satisfies the following properties:

- $s_1 = 1$
- s is *non-decreasing*
- For each n, the value of s_n is equal to the number of times that n appears in the sequence
- For each $n > 1$, the value of s_n is equal to the smallest integer which makes it possible to satisfy the previous conditions

You can check that its first few terms are:

$$1, 2, 2, 3, 3, 4, 4, 4, 5 \ldots$$

We see that 1 appears $s_1 = 1$ times, that 2 appears $s_2 = 2$ times, that 3 appears $s_3 = 2$ times, that 4 appears $s_4 = 3$ times... and so on.

Let's create a SpReadSheet with n rows and n columns, such that the cell in the ith row and jth column contains the value $s_i \times s_j$ (the product of the ith and jth entries in the SRS). For example, if $n = 4$, we get the following:

1	2	2	3
2	4	4	6
2	4	4	6
3	6	6	9

If you add up the values in each of the n^2 cells, what number do you get as a result? This number can get quite huge, so just output the remainder when this result is divided by the value 998244353.

You can check using the grid illustrated above that when $n = 4$, the answer is 64.

Subtasks

To be awarded the points for each subtask, answer the problem, given that subtask's value of n.

Subtask	Points	Constraints
1	**50**	$n = 15$
2	**50**	$n = 10^6$
3	**50**	$n = 10^{14}$
4	**50**	$n = 10^{18}$

Problem E
Sussy Spanning Trees

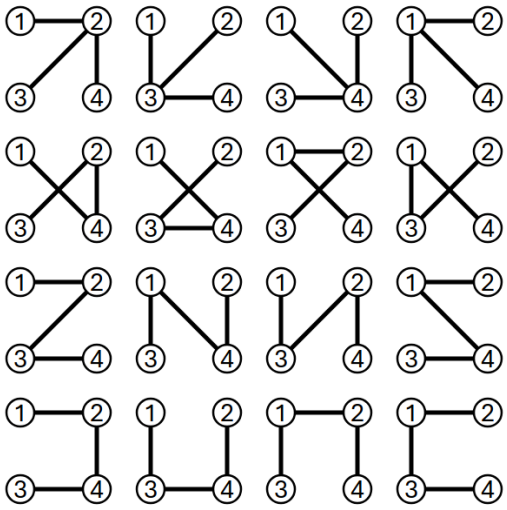
The Philippines' expedition to the moon takes place in a gossip-powered rocket ship. The ship has n rooms, each with a distinct name and purpose. The ship also has $n(n-1)/2$ two-way corridors, such that there is one corridor that *directly* connects each distinct pair of rooms in the ship. Each corridor can be uniquely identified by the two rooms at its endpoints.

May isang manlilinlang sa'tin. Having been warned of the presence of an impostor among their ranks, the ship's captain wants to choose some subset of corridors to **destroy**, subject to the following conditions:

- After destroying all the corridors in this subset, there should exist exactly *one* unique path (i.e. sequence of distinct corridors) that connects any two rooms in the ship.

This resulting configuration of corridors is called a *spanning tree*, and supposedly, this will make it easier to monitor the activities of all the Filipinauts on the ship.

Let $f(n)$ be equal to the number of different valid ways that the captain could choose a subset of corridors to destroy. For example, you can check that $f(4) = 16$. Here are the spanning trees formed from the remaining corridors, in all ways.



It was really nice of the captain to tell you about the term "spanning trees". Supposedly, there is *a lot of existing literature* about counting spanning trees that you might find *very useful*.

What is the sum of $f(n)$ across all integers n from 1 to N? This number can get quite huge, so just output the remainder when this result is divided by a given value m.

You can check that when $N = 4$ and $m = 11$, the answer is $(1 + 1 + 3 + 16) \bmod 11 = 10$.

Subtasks

To be awarded the points for each subtask, answer the problem, given that subtask's values of N and m.

Subtask	Points	Constraints
1	**50**	$N = 5$ and $m = 100$
2	**50**	$N = 10^6$ and $m = 2024696969$
3	**50**	$N = 10^{18}$ and $m = 693789912465408$
4	**50**	$N = 10^{18}$ and $m = 692062236180480$

Problem F
Digivisible

The *Terribly Advanced Mega Automaton* (TAMA) needs a new password, and multiplying two large prime numbers together just won't cut it anymore, for some reason. He wants to spice it up, but he is also lazy, despite being a robot.

After scouring his database of sequences for exactly `0.01 ms`, he now fancies *digivisible numbers* for his password. For you unenlightened mortals, a positive integer (with no leading zeros) is said to be digivisible if:

- All its digits belong to the set S.
- It is divisible by all its digits.

For example, if $S = \{2, 3, 5, 7\}$, then 777735 is digivisible.

TAMA wants your help in selecting digivisible numbers, but obviously he doesn't want you to know the one he actually ends up picking as the password for his super ultra secret project. So, given the set S and integers ℓ and r, please *count* the number of digivisible numbers from ℓ to r inclusive.

You can check that when $S = \{1, 5, 9\}$ and $\ell = 10^3$ and $r = 10^6$, the answer is 67.

Subtasks

To be awarded the points for each subtask, answer the problem, given that subtask's values of S and ℓ and r.

Subtask	Points	Constraints
1	**50**	$S = \{1, 2, 5\}$ and $\ell = 10^{17}$ and $r = 10^{18}$
2	**50**	$S = \{1, 2, 7\}$ and $\ell = 10^3$ and $r = 10^{16}$
3	**50**	$S = \{1, 3, 6, 9\}$ and $\ell = 10^6$ and $r = 10^{18}$
4	**50**	$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $\ell = 18^9$ and $r = 9^{18}$

Problem G
Supersetter

Literally one second after the *Terribly Advanced Mega Automaton* (TAMA) implemented his digivisible password, he was hacked by his ex-girlfriend-now-sworn-rival, the *Advanced Bipedal Kinetic Droid* (ABKD). He now realizes, in his infinite wisdom, that maybe it is not the mathematical properties of the password he uses that make it secure, but *how* his password is generated.

More than anything else, it just seems that TAMA is annoyed because ABKD was the one that had hacked his files. You tried suggesting that maybe he is to blame for her vendetta, because he did not put any effort into his past relationship. But he rebuffs you by saying that's 99.9% untrue. Also, he accuses you of victim-blaming, which is probably fair.

Tired of continuing this discussion, he shuts you up by tasking you with generating his new password using the so-called *supersetter process*. He is making you do it because, again, he is lazy. He's so lazy, in fact, that at this point, he doesn't care that you know his password.

What is the supersetter process? We start with some positive integer n that has no leading zeros. We produce a new number by the following method:

- *Partition* the digits of n (somehow) into contiguous subsegments.
- Then, preserving the order of those subsegments, duplicate each one. Concatenate these all (in order) and the result is the output of the supersetter process.

For example, let $n = 314159$. Here are some possible results of the supersetter process.

- If we partition it as `(31)(4)(159)`, then the result is `313144159159`.
- If we partition it as `(3)(1)(4)(1)(5)(9)`, then the result is `331144115599`.
- If we partition it as `(314159)`, then the result is `314159314159`.

Given an integer n, compute the sum of the results of this process across all 2^{s-1} possible ways to choose how to partition the digits of n (where s is the number of digits in n), then take the remainder when this result is divided by 998244353. This will be TAMA's new password; compute it.

You can check that when $n = 123$, the answer is 468912.

Subtasks

To be awarded the points for each subtask, answer the problem, given that subtask's value of n.

Subtask	Points	Constraints
1	**50**	$n = 2024$
2	**50**	$n = 1234567890$
3	**50**	$n = 11^{123}$
4	**50**	$n = 12^{123456}$

Problem H
Expected Value of Civil War

The *Advanced Bipedal Kinetic Droid* (ABKD) has a problem. She has many bones to pick with the *Terribly Advanced Mega Automaton* (TAMA), her horrid ex-boyfriend-now-sworn-archnemesis. But it is terribly unbecoming of a robot to engage in direct combat. How savage! Instead, she and TAMA battle it out by hiring human mercenaries to fight on their behalf. This is much more civilized.

There are n mercenaries for hire (where n is some even number). The combat rating of the ith mercenary is, coincidentally, equal to the ith prime number. For example, if $n = 4$, then the mercenaries' combat ratings are $[2, 3, 5, 7]$.

ABKD hacked into TAMA's file system—the first password was trivial for her to break, and the second one... you had directly leaked to her (Sorry, TAMA!). TAMA's files reveal that he hired *exactly* $n/2$ of the mercenaries to fight on his side. But despite TAMA's laziness, it seems he was careful enough to hide the names of *which* mercenaries he hired.

Undeterred, ABKD has a new backup plan. She will attempt to hire *all* the mercenaries. She is sure that the $n/2$ already hired by TAMA are loyal to him and will defect, but the $n/2$ that he *didn't* hire will remain on her side and fight on her behalf.

Because ABKD is meticulous, she wants you to help her analyze how the battle might go, across all possibilities. The *imbalance* of a scenario is computed as follows.

- Let T be the set of all mercenaries loyal to TAMA; let $\text{sum}(T)$ be the sum of their combat ratings.
- Let T' be the set of all mercenaries *not* loyal to TAMA, who are now loyal to ABKD; let $\text{sum}(T')$ be the sum of their combat ratings.
- Then, the imbalance of this scenario is equal to $(\text{sum}(T) - \text{sum}(T'))^2$.

For example, if $n = 4$ and TAMA's mercenaries have combat ratings $\{2, 5\}$, then ABKD's mercenaries will have combat ratings $\{3, 7\}$. The imbalance of this scenario is $((2 + 5) - (3 + 7))^2 = (-3)^2 = 9$.

There are $\binom{n}{n/2}$ possible ways for TAMA to have chosen his mercenaries (which then uniquely determines ABKD's mercenaries). Evaluate the sum of the imbalances across *all* these scenarios. This number can get quite huge, so just output the remainder when this result is divided by the value 998244353.

For example, if $n = 4$, then you can check that the answer is 118.

ABKD will compensate you beautifully after everything is over, and she is nothing if but a robot of her word.

Subtasks

To be awarded the points for each subtask, answer the problem, given that subtask's value of n.

Subtask	Points	Constraints
1	**50**	$n = 6$
2	**50**	$n = 20$
3	**50**	$n = 5000$
4	**50**	$n = 10^7$

Problem I
Stratosphere

Our friend Neumann Philips wanted to impress his rival, Princess Mattea Oleifera. She rebuked him one time, saying that there was still a lot he didn't know about life—naturally, Neumann understands this to mean that he is lacking sufficient background knowledge about Conway's Game of Life, a classic example of a Turing complete cellular automata. And, upon hearing that Conway's Game of Life was about "gliders and spaceships", Neumann decided the best way to learn about it was, of course, by skydiving.

The stratosphere can be modeled as a grid with r rows and c columns. Each square has a certain *air velocity* level determined entirely by its altitude. Specifically, all squares in the ith row from the bottom have an air velocity level of i.

Neumann, with only a paraglider, jumps out of a plane at the top-left-most cell, and his goal is to reach the bottom-right-most cell. In one "time step", he chooses between moving to the square directly to his right, or to the square directly below him (provided it exists). A collection of squares traversed in this manner (including the starting and ending squares) is called a *path*.

You can verify that each path contains $r + c - 1$ squares, and that there are $\binom{(r-1)+(c-1)}{r-1}$ different paths along the stratosphere. The *thrill* of a path is equal to the product of the air velocity levels in all the squares traversed in that path.

For example, if $r = 3$ and $c = 4$, then the stratosphere looks as follows. It has 10 different paths; one such path, whose thrill is $3 \times 3 \times 2 \times 1 \times 1 \times 1 = 18$, is highlighted below.

3	3	3	3
2	2	2	2
1	1	1	1

3	3	3	3
2	2	2	2
1	1	1	1

What is the sum of the thrills of all paths along the stratosphere? This number can get quite huge, so just output the remainder when this result is divided by the value 998244353.

You can check that when $r = 3$ and $c = 4$, the answer is 540.

Subtasks

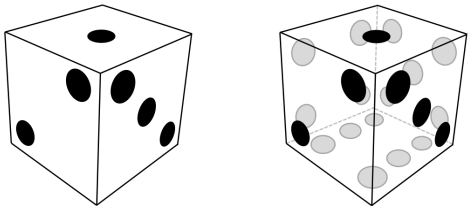
To be awarded the points for each subtask, answer the problem, given that subtask's values of r and c.

Subtask	Points	Constraints
1	**50**	$r = 4$ and $c = 6$
2	**50**	$r = 2024$ and $c = 2025$
3	**50**	$r = 2024$ and $c = 10^{18}$
4	**50**	$r = 10^6$ and $c = 10^{18}$

Problem J
Roll the Dice

Did you know that some pseudo-random generators out there use physical processes like radioactivity or lava lamps, in order to be as unpredictable (and close to truly random) as possible? Let's try doing that... but worse.

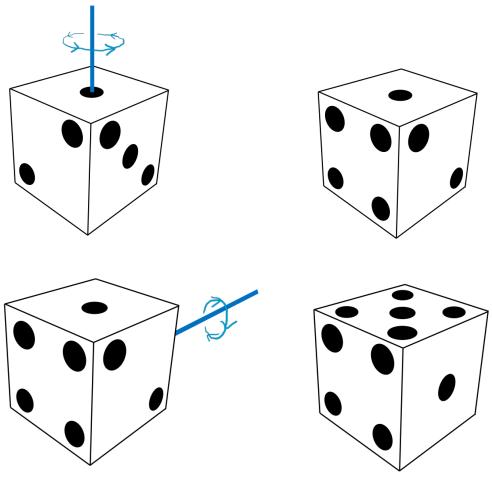
Consider a standard six-sided die. Like any die, the opposite faces add up to 7 (i.e. these pairs of faces are opposite from each other: 1 and 6; 2 and 5; and 3 and 4). Also, the 1 and 2 and 3 faces share a common vertex, arranged counterclockwise in that order around that vertex. This is sufficient to uniquely determine the layout of the die—see the following diagram to understand what the die looks like.



A single *right-hand twist around face x* is performed by doing the following:

- Do a thumbs up with your right hand. Then, place the base of your hand on face x, such that your pinky is touching the die and your thumb is jutting outwards.
- Rotate the die by 90°—the direction your fingers are curling is the direction in which you rotate the die. Note that the axis of rotation passes through faces x and $7 - x$.

In more jargon-y speak, perform a 90° rotation about the *normal vector* pointing out of face x, according to the right-hand rule. Here are concrete examples of performing a right-hand twist around face 1, followed by a right-hand twist around face 3.



Let $a_1, a_2, a_3, \ldots$ be an infinitely long sequence, where each term is an element from 1 to 6; we'll describe later how this sequence is generated. What matters is that we can use it—and the die, and an integer n—in order to generate a random integer from 1 to 6 via the following process:

- Consider all pairs of integers (ℓ, r) such that $1 \leq \ell \leq r \leq n$. There are $n(n+1)/2$ such pairs; choose one of them uniformly at random.
- Place the die on the table such that, initially, face 1 is facing upwards.
- Perform a right-hand twist around face a_ℓ, then a right-hand twist around face $a_{\ell+1}$, then a right-hand twist around face $a_{\ell+2}$... and so on, until finally you perform a right-hand twist around face a_r.
- The result of the process is the number on whichever face is facing upwards after all the twists have been performed.

Let $f(x)$ be the number of different pairs (ℓ, r) such that if this (ℓ, r) were chosen, then x would be the number generated by the process. Given the integer n, please calculate the remainder when the following value is divided by 998244353:

$$1f(1) + 2f(2) + 3f(3) + 4f(4) + 5f(5) + 6f(6)$$

To conclude, we describe how the sequence a is computed. First, let b_1, b_2, b_3, ... be a sequence, defined such that:

- $b_1 = 1$
- For $i > 1$, we let $b_i = (21\ b_{i-1} + 17) \bmod 2^{18}$

Then, we let $a_i = (b_i \bmod 6) + 1$.

You can check that when $n = 3$, the answer is 21.

Subtasks

To be awarded the points for each subtask, answer the problem, given that subtask's value of n.

Subtask	Points	Constraints
1	**50**	$n = 5$
2	**50**	$n = 2024$
3	**50**	$n = 10^6$
4	**50**	$n = 10^{18}$

Additional Resources

If you would like a physical die to aid you in visualizing the right-hand twists, here is a net of the die used in this problem. You can print it out and create your own paper die to play with!