National Olympiad in Informatics

TAMa

Contents

Important! Read the following:

- Unlike in other NOI.PH contests, your answer to each subtask is a single number, **not code.**
  - For each subtask, substitute in that subtask's value of n (or m, or whatever parameters) and solve the problem, given that value.
  - Type **only the final answer** into the input field for that subtask.
  - When making a submission, you may choose to answer only one subtask at a time (leaving the other fields blank). Your total score counts a subtask as solved if *any* previous submissions have solved that subtask.
- **Do not** put commas or spaces in numerical output. For example, if the problem is to compute 1111×1111, submit `1234321` **not** `1,234,321`.
- It is **highly recommended** to aim for 100 points in every problem before going for 150 or 200 points in most problems.
- Each problem has been designed according to a "one-minute rule".
  - We guarantee that for all subtasks of all problems, a sufficiently efficient algorithm exists that allows the answer to be obtained on a modestly-powered computer in **less than one minute**.
  - Of course, for particularly tricky problems, it may take you much longer than that to *come up* with the solution!
  - Although this rule will not be strictly enforced, we greatly encourage you to try and follow it! You'll learn new and interesting techniques if you choose to take this path, and we believe it is the most fun!
- You are encouraged to use the internet as a resource while solving these problems. We hope that in your research for this contest, you come across and learn many new topics in mathematics!
- The perfect score for this contest is 2023 points.
- Finding the problems too hard? Please join our upcoming **Abakoda 2023 Beginner Programming Contests** which start on November 11th! Stay tuned in our Discord for more details.
- Good luck and enjoy the contest! 😃

Problem A
Squares of Even Lucas Numbers

The Lucas sequence $L_0, L_1, L_2, L_3, L_4, L_5, \ldots$ is defined as follows. We let $L_0 = 2$ and $L_1 = 1$, then for $n \geq 2$, we let

$$L_n = L_{n-1} + L_{n-2}.$$

In other words, much like the Fibonacci sequence, each next term in the Lucas sequence is equal to the sum of the previous two terms. Its first few terms are:

$$2, 1, 3, 4, 7, 11, 18, 29, 47, 76, \ldots$$

Given n, let s be the sum of the **squares** of all even-valued terms of the Lucas sequence with index $< n$. What is the remainder when s is divided by m? (Also known as "$s \bmod m$")

For example:

- If $n = 9$, the sum is $s = 2^2 + 4^2 + 18^2 = 344$. Then, if $m = 5$, the answer is $(344 \bmod 5) = 4$.
- If $n = 100$ and $m = 2023$, then the answer is 272.

Subtasks

To be awarded the points for each subtask, answer the problem, given that subtask's values of n and m.

Subtask	Points	Constraints
1	**50**	$n = 30$ and $m = 1000$
2	**50**	$n = 10^6 + 2023$ and $m = 998244353$
3	**50**	$n = 10^{15} + 2023$ and $m = 8^8 + 2023$
4	**25**	$n = 10^{15} + 3000$ and $m = 10^{18} + 3000$
5	**25**	$n = 10^{15} + 2999$ and $m = 10^{18} + 2999$

Problem B
GCD of Fibonacci Subsets

The Fibonacci sequence $F_0, F_1, F_2, F_3, F_4, \ldots$ is defined as follows. We let $F_0 = 0$ and $F_1 = 1$, then for $n \geq 2$, we let

$$F_n = F_{n-1} + F_{n-2}.$$

In other words, much like the Lucas sequence, each next term in the Fibonacci sequence is equal to the sum of the previous two terms. An integer is called a *Fibonacci number* if it appears in this sequence. The first few **positive** Fibonacci numbers are:

$$1, 2, 3, 5, 8, 13, 21, \ldots$$

Consider the set S which consists of the first n positive Fibonacci numbers. If A is a non-empty subset of S, then we define $\gcd(A)$ (read as the **greatest common divisor** of A) to be the largest integer d such that every element of A is divisible by d. For example, if $n = 7$, then the first seven positive Fibonacci numbers are $\{1, 2, 3, 5, 8, 13, 21\}$, of which $\{2, 8\}$ is a non-empty subset, and $\gcd(\{2, 8\}) = 2$.

Given n, what is the sum of $\gcd(A)$ across **all** non-empty subsets A of S? The answer can get quite huge, so give the remainder when this result is divided by 998244353.

For example:

- if $n = 7$, the sum is 176;
- if $n = 30$, the sum is 1077269960, so the answer is

$$(1077269960 \bmod 998244353) = 79025607.$$

Subtasks

To be awarded the points for each subtask, answer the problem, given that subtask's value of n.

Subtask	Points	Constraints
1	**50**	$n = 12$
2	**50**	$n = 54$
3	**25**	$n = 10^4$
4	**25**	$n = 10^7$
5	**50**	$n = 11^{11}$

Problem C
Fruity Fractions

For some reason beyond our mortal understanding, the following math problem has gone viral and is now trending on the social media platform ξ dot com:

Only 99% of geniuses can solve this problem 😂😂😂

$$\frac{2}{🍎} + \frac{2}{🍎 + 2🥥} = \frac{3}{🍌} + \frac{1}{2🍎 - 🍌 + 3🥥}$$

🤧🤧🤧 **Can you find integer values for** 🍎, 🍌, 🥥?

This turns out to not be so hard. For example, (🍎, 🍌, 🥥) = (💯, 3️⃣, ➖4️⃣9️⃣) is one such solution, and you can find infinitely many more solutions.

Note that the denominators are nonzero in any valid solution.

For the actual task: Given a positive integer n, **count** the number of integer solutions (🍎, 🍌, 🥥) to the above problem, such that $\max(|🍎|, |🍌|, |🥥|) \leq n$ (note the use of absolute value functions). This number can get quite huge, so give the remainder when this result is divided by 998244353.

For example,

- if $n = 5$, the answer is 30;
- if $n = 10^5$, the answer is 679988514.

Do not submit your answer using emojis for digits.

Subtasks

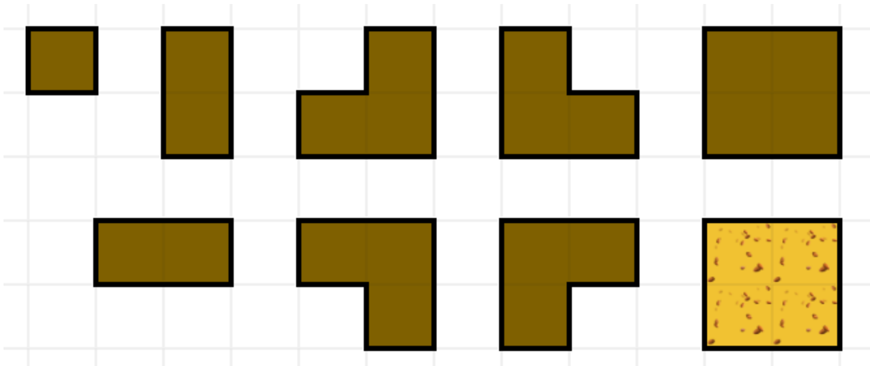
To be awarded the points for each subtask, answer the problem, given that subtask's value of n.

Subtask	Points	Constraints
1	**50**	$n = 3^5$
2	**50**	$n = 5^5$
3	**50**	$n = 5^{10}$
4	**50**	$n = 15^{10}$

Problem D
Sweet Tile O' Mine

Kelly Kawno is collaborating with famous YouTuber Mr Feast to expand into the chocolatiering business! They plan to release a line of chocolates called "Beast Bars," so named because you will feel like a **beast** after eating one!

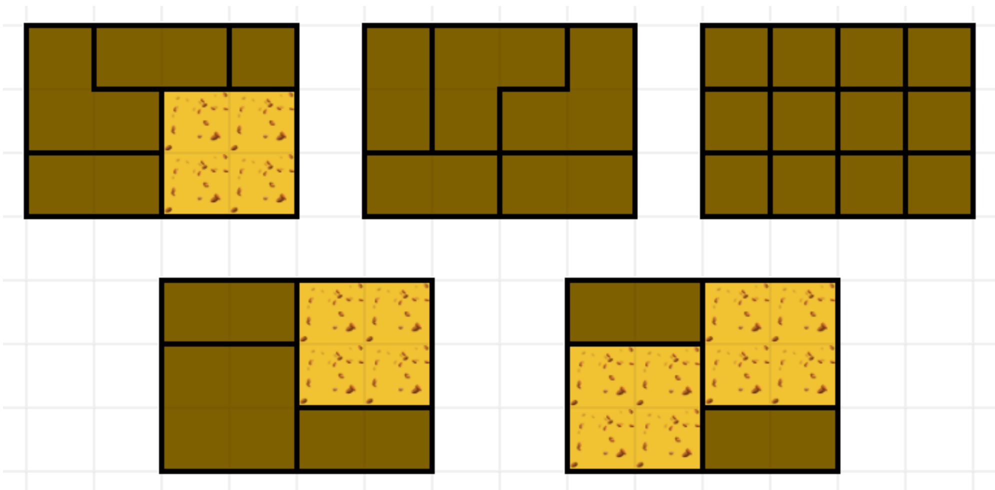
The Beast Bar shall be a rectangle consisting of r rows and c columns, split into $r \times c$ unit squares. Each bar is assembled by connecting together various *chocolate pieces* called "tiles" like a gigantic jigsaw puzzle. Here are the various kinds of tiles that Mr Feast can use!



Note that:

- *Rotations* of tiles are considered to be distinct from one another
- There are *two* different tiles whose shape is a 2×2 square—one made of milk chocolate, and one made with malt sugar extract.

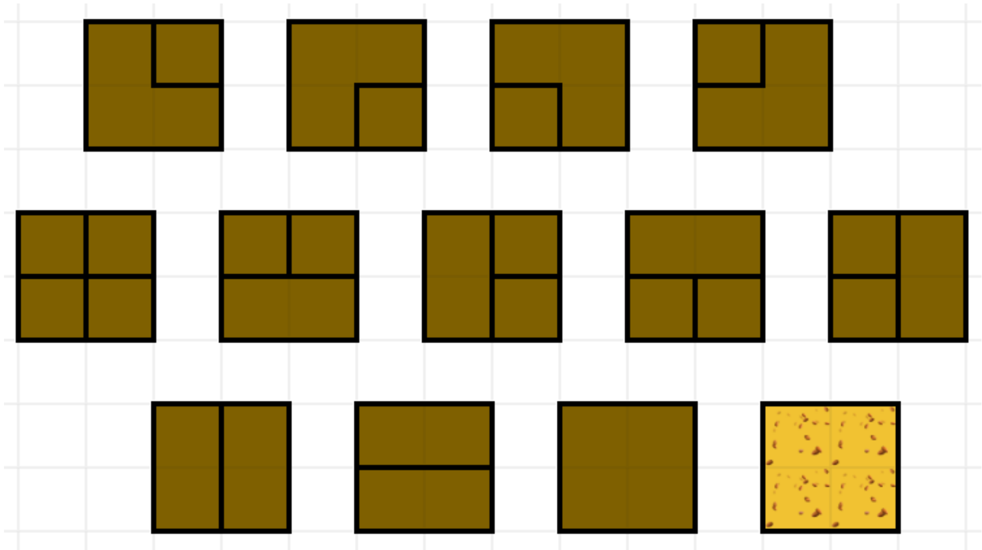
For example, here are five different valid ways to construct a Beast Bar when $r = 3$ and $c = 4$. Note that there must be no overlap between pieces, and no "extra" squares outside the rectangle.



How many different ways are there to construct an $r \times c$ Beast Bar? Two ways are considered different if there exists a square that belongs to a different kind of tile between the two ways. This number can get quite huge, so give the remainder when this result is divided by 998244353.

For example:

- if $r = 2$ and $c = 2$, the total count is 13. There are 13 ways to construct a 2×2 Beast Bar, as illustrated below:



- if $r = 3$ and $c = 4$, the answer is 4773;
- if $r = 7$ and $c = 7$, the answer is 297815281.

Subtasks

To be awarded the points for each subtask, answer the problem, given that subtask's values of r and c.

Subtask	Points	Constraints
1	**50**	$r = 2$ and $c = 8$
2	**50**	$r = 3$ and $c = 10^6$
3	**25**	$r = 3$ and $c = 10^{18}$
4	**25**	$r = 7$ and $c = 10^6$
5	**25**	$r = 7$ and $c = 10^{18}$
6	**25**	$r = 13$ and $c = 10^{18}$

Problem E
The Glass Maze

Pusit Bulaga, the Philippines' most popular noontime variety show, has hired you as a consultant to help analyze the game in their brand new segment, *the glass maze*. The glass maze can be represented as a square grid with n rows and n columns. Let (i, j) represent the cell in the ith row from the top and jth column from the left, where $1 \le i \le n$ and $1 \le j \le n$.

A panel of glass is placed in each cell. The glass panel in cell (i, j) can hold a weight of **at most** $b_{i,j}$ milligrams; any more than that, and the panel breaks.

For example, if $n = 4$, then the maze has 4 rows and 4 columns, and the glass maze could be represented by the grid with the following values:

2000023	55500000	55500000	75000000
60000000	86236000	42000000	87000000
75000000	90000000	50000000	5000000
999999999	40000000	60000000	87000000

In this grid, $b_{1,1} = 2000023$, $b_{1,2} = 55500000$, and so on.

A contestant, holding all their treasure, begins the challenge by being placed on some cell (i_s, j_s). Their objective is to safely travel to cell (i_t, j_t) in order to escape. In one move, a contestant can jump to any of the cells directly north, south, east, or west from their current cell (provided such a cell exists).

If at any time a contestant is on a glass panel (*including* the ones at (i_s, j_s) and (i_t, j_t)) and their weight exceeds the weight limit of that glass panel, then that panel breaks and the contestant plummets to their doom, losing all their treasure.

For a given grid, let $W(i_s, j_s, i_t, j_t)$ be defined as the *maximum* weight w (in milligrams) such that it is possible for a person with weight w to successfully travel from (i_s, j_s) to (i_t, j_t) via some path. For example, for the grid above, you can check that $W(1, 4, 4, 3) = 50000000$, as illustrated below.

2000023	55500000	55500000	75000000
60000000	86236000	42000000	87000000
75000000	90000000	50000000	5000000
999999999	40000000	60000000	87000000

But that grid is just an example. The *actual* values of $b_{i,j}$ for the real game are generated by the game designers via the following formula:

$$b_{i,j} = 1669661^{2023\times i\times j} \bmod 998244353,$$

where again, i and j both range from 1 to n. For example, if $n = 2$, then the panels' weight limits look like this:

287271328	149369469
149369469	817235580

Given n, evaluate the sum of $W(i_s, j_s, i_t, j_t)$ across **all** tuples (i_s, j_s, i_t, j_t) such that $1 \le i_s, i_t, j_s, j_t \le n$ and $(i_s, j_s) \neq (i_t, j_t)$; there are $n^4 - n^2$ such tuples. This number can get quite huge, so give the remainder when this result is divided by 998244353.

For example:

- For $n = 2$, the sum is 1792433628, so the answer is

$$(1792433628 \bmod 998244353) = 794189275.$$

- For $n = 5$, the sum is 188167059380, so the answer is

$$(188167059380 \bmod 998244353) = 497121016.$$

Subtasks

To be awarded the points for each subtask, answer the problem, given that subtask's value of n.

Subtask	Points	Constraints
1	**50**	$n = 4$
2	**50**	$n = 12$
3	**25**	$n = 143$
4	**25**	$n = 3141$
5	**50**	$n = 3^{3^3}$

Problem F
Hit the Griddy

Suppose we have been tasked with creating a compilation grid of *memes* with r rows and c columns. In each of the rc cells, you can either put a **cool** meme or a **cringe** meme. If coolness and cringeness are the only things that matter in life, then that means there are 2^{rc} possible compilation grids.

As we all know, two cringe memes cancel out and become cool again. Thus, a compilation grid G is called **based** if it satisfies both of these conditions:

- There are an even number of cringe memes in every row.
- There are an even number of cringe memes in every column.

Given an arbitrary compilation grid G, let $B(G)$ be the minimum number of memes that need to be transformed (either from cringe to cool, or cool to cringe) such that G becomes based. Note that if G is *already* based, then $B(G) = 0$.

For example, let 🗿 indicate a cool meme, and let 😬 indicate a cringe meme. If $r = 3$ and $c = 4$, then one possible compilation grid G is

😬😬😬🗿
😬🗿😬🗿
🗿😬🗿😬

This G is not already based, but can be made based by changing only one meme (turn the top-rightmost meme from cool to cringe); thus, $B(G) = 1$.

Given r and c, output the sum of $(B(G))^3$ across all 2^{rc} possible compilation grids G that have r rows and c columns. This number can get quite huge, so give the remainder when this result is divided by 998244353.

For example,

- if $r = 3$ and $c = 5$, then the answer is 858624;
- if $r = 6$ and $c = 9$, then the answer is 419465649.

Subtasks

To be awarded the points for each subtask, answer the problem, given that subtask's values of r and c.

Subtask	Points	Constraints
1	**50**	$r = 5$, $c = 5$
2	**50**	$r = 64$, $c = 100$
3	**25**	$r = 4000$, $c = 5000$
4	**25**	$r = 6400000$, $c = 6400000$
5	**50**	$r = 64000000$, $c = 10^{16}$

Problem G
The Amazons' Primes

Maxi and Mini are twin warrior sisters who also recreationally indulge in mathematics. Specifically, both of them love prime numbers, but in very different ways—Maxi likes large primes, whereas Mini likes little primes.

Suppose both sisters start with a shared positive integer sequence $[a_1, a_2, \ldots, a_n]$ whose elements are all greater than 1.

Maxi generates the **Maxi-sequence** $[M_1, M_2, \ldots, M_n]$ by replacing each element of a with its *largest* prime factor, i.e.,

$$M_i = \text{largest prime that divides } a_i$$

for each i from 1 to n. Similarly, Mini generates the **Mini-sequence** $[m_1, m_2, \ldots, m_n]$ by replacing each element of a with its *smallest* prime factor, i.e.,

$$m_i = \text{smallest prime that divides } a_i$$

for each i from 1 to n.

Given integers n and b, we say a sequence of n integers $[a_1, a_2, \ldots, a_n]$ is **Amazing** if it satisfies all of the following conditions:

- $1 < a_i < b$ for all i from 1 to n;
- Its corresponding Maxi-sequence M is *strictly* increasing;
- Its corresponding Mini-sequence m is also *strictly* increasing.

How many Amazing sequences are there? This number can get quite huge, so give the remainder when this result is divided by 998244353.

For example:

- if $n = 4$ and $b = 400$, then $[256, 45, 385, 169]$ would be one Amazing sequence, and there are 60834149 such Amazing sequences overall;
- if $n = 3$ and $b = 18$, then there are 169 Amazing sequences;
- If $n = 69$ and $b = 420$, then there are 4681408354305143955 Amazing sequences, so the answer is

$$(4681408354305143955 \bmod 998244353) = 968682385.$$

Subtasks

To be awarded the points for each subtask, answer the problem, given that subtask's values of n and b.

Subtask	Points	Constraints
1	**50**	$n = 10,\ b = 48$
2	**50**	$n = 100,\ b = 4000$
3	**50**	$n = 2400,\ b = 300000$
4	**50**	$n = 80000,\ b = 12000000$

Problem H
Pillage Twilight Heroes

Pillage Twilight Heroes is an all-new multiplayer game, and it's completely free! Throughout the game, you have to collect weapons to defeat increasingly strong enemies in round-based combat. Every weapon has an associated power rating (a positive integer) that determines how much damage it can deal every round.

Starting off with no weapons, a party of n players plans to use the promo code "PILLAGE999" to get w weapons, each with a power rating chosen independently and uniformly at random from the integers from 1 to k. Different weapons may get the same power rating. From here, each player must select a weapon, but no two players may pick the same one.

The players in the party are smart and will pick the n weapons with the highest power ratings (if there is a tie for the nth strongest weapon, the players break the tie arbitrarily by picking their favorite of those weapons).

For example, suppose $n = 2$, $w = 6$, and $k = 3$.

- After inputting the promo code, we generate $w = 6$ weapons with random power ratings from 1 to $k = 3$; one *possible* outcome would be weapons with power ratings $[2, 3, 1, 2, 2, 1]$.
- The party of $n = 2$ picks the two strongest weapons—which in the case of this outcome, would be the weapon with power rating 3, and then any of the weapons with power rating 2. The sum of the party's weapons' power ratings would be $3 + 2 = 5$.
- If we were to repeat this process many many times, we find that on average, we should **expect** the sum of the party's weapons' power ratings to be $4040/729 \approx 5.5418$.

The developers of Pillage Twilight Heroes don't want the game to be too easy or too hard, so they ask you a simple question. You are given n, w, and b. On average, what do we **expect** to be the value of the sum of the party's weapons' power ratings, assuming that the weapons were generated and then selected via the described process? It can be shown that this number is rational; output this number "mod 998244353". (What does this mean? See the **Notes** below for an explanation of what this means.)

Here are some examples:

- If $n = 2$, $w = 3$ and $k = 6$, the average is $203/24$. It turns out that

$$(203/24 \bmod 998244353) = 207967582,$$

so that's the answer.

- If $n = 2$, $w = 6$ and $k = 3$, the average is $4040/729$. It turns out that

$$(4040/729 \bmod 998244353) = 191706740,$$

so that's the answer.

Subtasks

To be awarded the points for each subtask, answer the problem, given that subtask's values of n, w and k.

Subtask	Points	Constraints
1	**50**	$n = 5,\ w = 50,\ k = 5$
2	**50**	$n = 200,\ w = 500,\ k = 300$
3	**25**	$n = 600,\ w = 10^8,\ k = 500$
4	**25**	$n = 9000,\ w = 10^{16},\ k = 9000$
5	**50**	$n = 2500000,\ w = 10^{16},\ k = 500000$

Notes

What does a rational number modulo a prime mean? Let r be a rational and p be a prime. We define r **modulo** p to be the unique integer r' such that $0 \le r' < p$ and the rational number $r - r'$ is "divisible by p". (Sometimes, this doesn't exist, but it can be shown that it does for the answers to this problem.)

What does it mean for a rational number to be divisible by a prime? We say that a rational number **is divisible by** p if and only if it can be expressed as a/b for integers a and b, where a is divisible by p, but b is not.

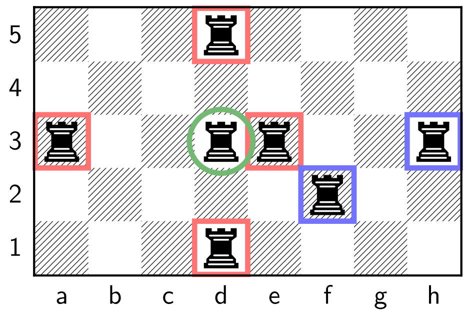
For example, $35/14$ and $150/77$ are divisible by 5, but $7/8$ and $3/10$ are not. Also, $(16/3 \bmod 5) = 2$ because $16/3 - 2 = 10/3$, and $10/3$ is divisible by 5.

Problem I

The Idea is to Sacrifice THE ROOOOOOOK

Consider a variation of chess which takes place on a grid with r rows and c columns (and thus rc *cells*). There are no "colors", and the only piece is the *rook*. A rook can move any number of steps horizontally or vertically without "jumping over" other pieces, and may capture any piece in its path. If a rook can capture some piece, we say that this rook *threatens* that piece.

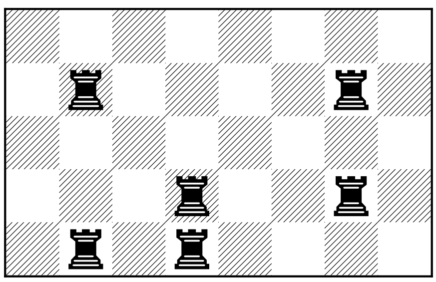
For example, in the following board where $r = 5$ and $c = 8$, the rook at d3 (encircled green) threatens the rooks at d5, e3, a3, and d1 (boxed red), but does *not* threaten the rooks at f2 or h3 (boxed blue).



We say a chessboard consisting of rooks is **unstable** if there exists a non-negative integer d such that:

- For every rook, there are exactly d other rooks in the same row as it that threaten it.
- For every rook, there are exactly d other rooks in the same column as it that threaten it.

For example, the following chessboard with $r = 5$ rows and $c = 8$ columns is unstable (taking $d = 1$):



There are many such chessboards; in fact, there are exactly 168461 unstable chessboards with $r = 5$ rows and $c = 8$ columns.

Given n, what is the number of unstable chessboards with exactly n cells? This number can get quite huge, so give the remainder when this result is divided by 998244353.

For example,

- if $n = 66$, then the number of unstable chessboards is 76683692;
- if $n = 100$, then the number of unstable chessboards is 3239590777554, so the answer is

$$(3239590777554 \bmod 998244353) = 287852069.$$

Two chessboards are considered the same iff they have the same number of rows, the same number of columns, and the same locations of rooks. (Each cell can only be empty or contain one rook.) Also, note that rotations and reflections are considered distinct.

Subtasks

To be awarded the points for each subtask, answer the problem, given that subtask's value of n.

Subtask	Points	Constraints
1	**50**	$n = 77$
2	**50**	$n = 3000$
3	**25**	$n = 30^3$
4	**25**	$n = 42^5$
5	**25**	$n = 9^{15}$
6	**25**	$n = 10^{15}$

Problem J
Immortal Plans

It is the beginning of January 1, 4000, and the immortal witch Bernkastel desperately wants to cure her boredom. She has watched every TV show created by humanity—listened to every song, played every game, read every book, and committed every crime (she doesn't recommend this last one). In the end, she always becomes bored with whatever new thing she tries to get into.

You point out that maybe her boredom stems from the fact that she binges everything, which leads to her easily getting sick of them. Perhaps if she *spaced out her activities* better, then she would still be able to find excitement in them when she comes back to repeat them. Intrigued, she asks you to develop a plan for her.

- Every day, you must choose an activity for her to do. Suppose there are c different activities to choose from for each day.
- A "plan" consists of specifying which activity she should do, *every single day*, for the next n years.
  - This plan will last for the rest of her immortal life—after the nth year, she cycles back to the first day of the first year of the plan... and so on.
  - **For the easier version of this problem**, we consider a universe where leap years do not exist. For full points, you must handle leap years. See the notes for a full explanation on the structure of a year.
- For a plan to be *properly-spaced*, it must satisfy the following constraints:
  - For any two consecutive days in Bernkastel's infinite life, it is never the case that the activities performed on each day are the same.
  - For any two consecutive *months* in Bernkastel's infinite life, it is never the case that the activities performed on the *first day* of each month are the same.

For example, suppose $n = 400$ and $c = 3$, and leap years exist. Let's label the three activities 1, 2, and 3. Here are the beginnings of **one** possible plan:

- For January 1 to 31, 4000, we may choose activities in the pattern $1, 2, 3, 2, 1, 2, 3, 2, \ldots, 1, 2, 3$.
- For February 1, 4000, activity 3 was chosen on the previous day (January 31) whereas activity 1 was chosen on the first day of the previous month (January 1); therefore, we are forced to pick activity 2 for this day.
  - Note that 4000 would be a leap year in a universe where those exist.
  - For February 1 to 29, 4000, we may choose activities in the pattern: $2, 3, 2, 3, 2, 3, \ldots, 2, 3, 2$.

- Repeat this, choosing the activities of every day of every month, until December of 4399.

Because $n = 400$, the cycle repeats after 400 years, and we have **no more choices to make**: the month of January 4400 will again follow the pattern $1, 2, 3, 2, 1, \ldots, 1, 2, 3$; then February 4400 will again follow the pattern $2, 3, \ldots, 2, 3, 2$; ...and so on, for the rest of time. Note that because of this, it is important to guarantee that neither December 1, 4399 nor December 31, 4399, have activity 1 selected, because January 1 (in the year 4400) comes after.

Let $P(n, c)$ count the number of properly-spaced plans if there are c activities to choose from for each day for the next n years.

For example, again consider $n = 400$ and $c = 3$.

- In the universe where leap years **don't** exist,

$$P(400, 3) \bmod 9982443520000001 = 6294703839080434.$$

- In the universe where leap years **do** exist,

$$P(400, 3) \bmod 9982443520000001 = 1601108114453425.$$

These values are gigantic, which is why we only gave them modulo a large prime.

Given n and C, find the sum of $P(n, c)$ across all integers c from 1 to C. Only give the remainder when this sum is divided by 9982443520000001. We remind you that here, the first year of the plan happens in the year 4000.

For example, suppose $n = 400$ and $C = 4$. Then the required answer is

$$(P(400, 1) + P(400, 2) + P(400, 3) + P(400, 4)) \bmod 9982443520000001;$$

in a universe where leap years **do not exist**, the answer is 7308545946151460, and in a universe where leap years **do exist**, the answer is 8403801211504682.

Subtasks

To be awarded the points for each subtask, answer the problem, given that subtask's values of n and C, and whether or not leap years exist.

Subtask	Points	Constraints
1	**50**	Leap years do not exist, $n = 1$ and $C = 3$
2	**25**	Leap years do not exist, $n = 4$ and $C = 10$
3	**25**	Leap years exist, $n = 400$ and $C = 10$
4	**25**	Leap years exist, $n = 400$ and $C = 2023$
5	**25**	Leap years exist, $n = 400$ and $C = 10^{18}$
6	**25**	Leap years exist, $n = 2000$ and $C = 10^{18}$
7	**25**	Leap years exist, $n = 10000$ and $C = 10^{18}$

Notes

A **normal calendar year** has 365 days split into 12 months. The 12 months are, in order: January, February, March, April, May, June, July, August, September, October, November, December. Each month consists of some number of days.

- These months have 31 days: January, March, May, July, August, October, December.
- These months have 30 days: April, June, September, November.
- These months have 28 days: February.

A **leap year** has 366 days, with the only change from a normal calendar year being that February has 29 days instead. For subtasks 1 and 2, there are no leap years. For subtasks 3 through 7, they are determined by the following rules:

- By default, we assume that a year is not a leap year.
- But, if a year is divisible by 4, it is a leap year.
- As an exception to this rule, if a year is divisible by 100, it is *not* a leap year.
- As an exception to the exception, if a year is divisible by 400, it *is* a leap year.

For example, the following would be leap years: 4000, 4028, 123456, and 777200; and the following would *not* be leap years: 4321, 5000, 20232023, and 99999900.

For subtasks 3 through 7, the given n is divisible by 400, in order for the problem to make sense.

Bonus Problem
sTANDARD eTHICAL aMAT

Nwv whk, n ethical wn 'vy snwj'w pn-ratohrh. Rnwn'o ln-awzho lh 'dh!

Dfa standard uhllhw vymyrht ht suv hg dkh ehrydyma ywdasatr l nwv m yr n uhllhw vymyrht hg l nwv m dfnd yr standat dfnw nwo hdfat uhllhw vymyrht hg l nwv m. Eht azalech:

- dfa suv hg 8577 nwv 8577 yr 8577;
- dfa suv hg 155 nwv 1071 yr 75;
- dfa suv hg 7171 nwv 71500 yr 5;
- dfa suv hg 928986 nwv 892396 yr 8;
- dfa suv hg 76 nwv 58 yr 76.

Wand 'wh?

Gjw gnud: Yw Znenwara yd yr unccav "rnyvny phjonpjrjj"; rn Phtanw wnln'o "ufhavnashwsonprj". Rn pnrnlnnws encnv, entnws kncn r'onws Dnsnchs DhD. 'Ens dywnwhws lh wnlnw ry Shhsca, "eywnpnlncnpyws pntnwyknws vymyrht" 'ojws rnrniyfyw w'on. Entnws eycyd cnws 'vy in? RDAL ucnrrar yw EF dawv dh ia dnjsfd yw Awscyrf, ht nd iard Dnscyrf (ht 'ojws tasyhwnc cnwsjnsa) kydf daufwyunc khtvr jwdtnwrcndav. Yd tancco lnpar ohj gaac cypa n lncnwrnws gyrf!

Nwokno, tysfd, inup dh ethical. Whk, ka rno dfnd n dtyecad hg ehrydyma ywdasatr (n, o, i) yr *pndnpndnpn* yg yd rndyrgyar ncc hg dfa ghcchkyws:

- dfa suv hg n nwv o yr i;
- dfa vauylnc taetarawdndyhw hg n hwco jrar dfa vysydr 8, 1 nwv 6, nwv anuf hg dfhra neeantr nd canrd hwua;
- dfa vauylnc taetarawdndyhw hg o hwco jrar dfa vysydr 3, 5 nwv 7, nwv anuf hg dfhra neeantr nd canrd hwua;
- dfa vauylnc taetarawdndyhw hg i hwco jrar dfa vysydr 4, 9 nwv 2, nwv anuf hg dfhra neeantr nd canrd hwua.

Ght hjt ejtehrar, dfa vauylnc taetarawdndyhw hg n ehrydyma ywdasat yr jwyqja nwv vharw'd fnma n canvyws bath.

Nlhws ncc dtyecar (n, o, i) dfnd nta pndnpndnpn, kfnd yr dfa lywyljl ehrryica mncja hg $n \cdot o \cdot i$?

Idk ihwjr cnws 'dhws ethical wn 'dh! Zjrd ght gjw :E Dfyr ethical yr hwco khtdf 25 ehywdr. Ijd rdycc, pjws ln-rhcma lh r'on, ether dh ohj. Sjvcnp! Nwv yg ohj'ta nica dh tanv dfyr, ecanra rno yw dfa WHY ratmat dfnd dfyr ethical yr etaddo rdnwvntv (nwv adfyunc tyw r'oaleta)!