Experiment! AMOGUS Edition

First, let's try to be concrete about what exactly the problem is asking from us, in precise mathematical language. Knowing our "win condition" might give us direction in what to try out.

Suppose we call k emergency meetings (for this problem, $k = 18$). Each meeting involves some selection of players.

When do we lose? If player i is the witness, and player j is the killer, then we must call a meeting where i is present but j is not, in order for i to talk. Conversely, if j attends every meeting that i does, then we lose.

But the judge in this problem is ~~a cheater~~ adaptive. If there exists *any* pair of players i and j such that j attends every meeting that i does, then the judge could always claim that i is the witness and j is the killer.

Conversely, though, if we can ensure that for every ordered pair (i, j), there exists a meeting that i attended but j didn't, then the judge is forced to select a witness and have them talk. Then, we win!

Let's phrase things more formally, using mathematical objects. For the ith player, let S_i be the set of meetings that they attended. If j attended every meeting that i did, then in the language of sets, we'd say that $S_i \subseteq S_j$.

Therefore, in order to win, we must be able to produce a collection of n subsets of the k meetings, such that no subset is a subset of any other subset (what a mouthful...)

There turns out to be an infuriatingly straightforward solution. If two different subsets have the same size, then it is impossible for one to be a subset of the other. For some fixed value of s, there are exactly $\binom{k}{s}$ subsets of size s, and this is known to maximized when $s = \lfloor k/2 \rfloor$. With $m = 18$, we can accommodate a maximum of $\binom{18}{9} = 48620$ players, which is just enough for 100 points in this problem!

Remark: With more advanced mathematics, we can prove something stronger: $\binom{k}{\lfloor k/2 \rfloor}$ is actually the *best possible result*, as per Sperner's theorem.