Canonizing Cannonade

It's always good to start by playing around with the problem so that we can find some *necessary* conditions.

- It is necessary that $m \le k \times \max(r, c)$.
  - Each cannonball can only hit at most $\max(r, c)$ soldiers per shot, so if m is too big, then it would be literally impossible to clear all of them in k shots.
- It is necessary that $k \le m$.
  - Note that a solution always exists which hits all soldiers in m or fewer cannonballs—target each soldier one by one. But the resiliency is the *minimum* number of cannonballs needed, and if $m < k$, then this would contradict the minimality of k.
- It is necessary that $k \le \min(r, c)$.
  - Similar to the previous argument, note that a solution always exists which hits all soldiers in $\min(r, c)$ or fewer cannonballs—target each row (or column) one-by-one. But if $\min(r, c) < k$, then this would contradict the minimality of k.

These are all necessary for a solution to exist; if any of them are not met, then the task is impossible. Wouldn't it be nice if they were also *sufficient*? That is, it would be nice if whenever these conditions hold, a solution always exists. We now have concrete direction—let's try to prove that our wishful thinking is true!

Without loss of generality, assume $r \le c$, i.e. the rows are longer. If not, swap the two values, and then just transpose the grid we get in our answer.

First, place a soldier in each of `grid[i][i]` for i from 0 to $k-1$ ("along the main diagonal"). This is always possible because $k \le m$ (so we have enough soldiers) and $k \le r$ (so we have enough room). This ensures that the resiliency must be at least k, because we need at least k cannonballs just to hit these soldiers (since each of their rows and columns is distinct from one another, it is impossible to hit two of them in one shot).

Then, "fill up the rows" with soldiers, starting with the first row and going top-to-bottom, until m soldiers have been placed on the grid. We will never need more than the first k rows, because $m \le kc$. This ensures that the resiliency must be at most k, because a solution exists that hits all soldiers using only k cannonballs (sending one cannonball to each row).

Because our construction has a resiliency that is at least k and at most k, we conclude that its resiliency is exactly equal to k, which is what we wanted!

Here is an example of what that might look like, for $r = 6$, $c = 7$, $k = 5$, and $m = 25$.

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#######
#######
#######
##.#...
....#..
.......
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Note that in the above example, it would still be possible to squeeze up to $m = 35$ soldiers in.

Remark: Consider the "checker" needed for this problem—Given a formation, determine the minimum number of cannonballs needed in order to hit all the soldiers. It turns out that this problem is *significatly* more difficult, and it requires a heavy-duty algorithm in order to be solved!

For those interested, the prerequisite topics are *minimum vertex cover* and *maximum bipartite matching*.