National Olympiad in Informatics

TAMa Practice

Contents

Notes

- It is **highly recommended** to aim for 100 points in every problem before going for 150 or 200 points in most problems.
- Each problem has been designed according to a "one-minute rule".
  - We guarantee that for all subtasks of all problems, a sufficiently efficient algorithm exists that allows the answer to be obtained on a modestly-powered computer in **less than one minute**.
  - Of course, for particularly tricky problems, it may take you much longer than that to *come up* with the solution!
  - This rule will not be strictly enforced, but we hope that it gives you an idea of what we expect a decent running time to be like, and also the motivation to improve your implementation if your program takes much longer than that to execute.
- You are encouraged to use the internet as a resource while solving these problems. We hope that in your research for this contest, you come across and learn many new topics in mathematics!
- Good luck and enjoy the contest!

Problem A
Sum of Sum of Odds

Let $f(n)$ be the sum of the first n positive odd numbers, i.e.,

$$f(n) = 1 + 3 + 5 + \cdots + (\text{the } n\text{th positive odd number}).$$

For example, $f(4) = 1 + 3 + 5 + 7 = 16$.

What is the sum of $f(n)$ across all integers n from 1 to N? This number can get quite huge, so give the remainder when this result is divided by $10^6 + 37$.

For example:

- If $N = 5$, the answer is 55.
- If $N = 200$, the answer is 686626.

Subtasks

Answer the problem for each value of N to get that subtask's corresponding points.

Subtask	Points	Constraints
1	**50**	$N = 10$
2	**50**	$N = 10^4$
3	**50**	$N = 3^{15}$
4	**50**	$N = 2023 \times 10^{14}$

Notes

We explain here why the answer is 55 when $N = 5$.

- $f(1) = 1$
- $f(2) = 1 + 3 = 4$
- $f(3) = 1 + 3 + 5 = 9$
- $f(4) = 1 + 3 + 5 + 7 = 16$
- $f(5) = 1 + 3 + 5 + 7 + 9 = 25$

Thus, the answer is $1 + 4 + 9 + 16 + 25 = 55$.

Problem B
Mod Powers

NOI.PH has appointed Alice as a new moderator for their Discord server. Using her new mod powers, she wants to create a super-secret password of length n, subject to the following requirements:

- All n characters are lowercase English letters;
- She should **not** be able to find two adjacent letters in her password such that both letters are vowels, or both letters are consonants.

For the purposes of this problem, `y` is always treated as a consonant.

For example, for $n = 6$, the strings `alexis` and `yugito` and `hehehe` are possible passwords, but the strings `aisuru` and `dynamo` and `aaaaaa` are **not**.

How many possible passwords of length n are there? This number can get quite huge, so give the remainder when this result is divided by $10^9 + 7$.

For example:

- If $n = 3$, the answer is 2730.
- If $n = 6$, the answer is 2315250.
- If $n = 169$, the answer is 163737331.

Subtasks

Answer the problem for each value of n to get that subtask's corresponding points.

Subtask	Points	Constraints
1	**50**	$n = 10$
2	**50**	$n = 13^7$
3	**50**	$n = 10^{18}$
4	**50**	$n = 7^{7^{7^{2023}}}$

Problem C
Summer SEMs

Suppose you have a sequence $[v_1, v_2, \ldots, v_m]$. The **total square error** of some number x with respect to this sequence is given by the summation

$$\sum_{i=1}^{m} (x - v_i)^2.$$

In words: for each term, find its difference from x and then square the result; then, sum up these results across all terms. For example, the total square error of 4 with respect to the sequence $[2, 3, 5]$ is $(4-2)^2 + (4-3)^2 + (4-5)^2 = 6$.

The **square-error minimizer** of some sequence is the real number that *minimizes* the total square error with respect to that sequence. It can be shown that the square-error minimizer of any non-empty sequence is unique. For example, the square-error minimizer of $[2, 3, 5]$ is the number $3.333\ldots$.

Now, define the function $\text{SumSEMs}(a)$ on a sequence $a = [a_1, a_2, \ldots, a_n]$ as follows. Consider this operation, which can be done on a contiguous subsequence of a.

- Let ℓ and r be integers such that $1 \leq \ell \leq r \leq n$.
- Find the square-error minimizer of $[a_\ell, a_{\ell+1}, \ldots, a_r]$, the **contiguous** sequence of elements which starts at index ℓ and ends at index r.

Let $\text{SumSEMs}(a)$ be the sum of the square-error minimizers found by this operation across **all** values of ℓ and r such that $1 \leq \ell \leq r \leq n$. It can be shown that $n! \times \text{SumSEMs}(a)$ is always an integer. For example, $\text{SumSEMs}([2, 3, 5, 7]) = 505/12$, and indeed $4! \times (505/12) = 1010$, which is an integer.

Define the sequence $a = [a_1, a_2, \ldots, a_n]$ as follows. Let $a_1 = 2023$, and for $2 \leq k \leq n$, let

$$a_k = (2024 \times a_{k-1} + 1521) \bmod 998244353.$$

For example, if $n = 4$, then $a = [2023, 4096073, 304498449, 388096496]$.

What is the value of $n! \times \text{SumSEMs}(a)$? The answer can get quite huge, so give the remainder when this result is divided by 998244353.

For example:

- If $n = 3$, the answer is 412117155.
- If $n = 50$, the answer is 723001699.

Subtasks

Answer the problem for each value of n to get that subtask's corresponding points.

Subtask	Points	Constraints
1	**50**	$n = 5$
2	**50**	$n = 1600$
3	**50**	$n = 16000$
4	**50**	$n = 16 \times 10^6$

Notes

We explain here why $\text{SumSEMs}([2,3,5,7]) = 505/12$.

- Taking $\ell = 1$ and $r = 1$, the square-error minimizer of $[2]$ is 2.
- Taking $\ell = 2$ and $r = 2$, the square-error minimizer of $[3]$ is 3.
- Taking $\ell = 3$ and $r = 3$, the square-error minimizer of $[5]$ is 5.
- Taking $\ell = 4$ and $r = 4$, the square-error minimizer of $[7]$ is 7.
- Taking $\ell = 1$ and $r = 2$, the square-error minimizer of $[2,3]$ is $5/2$.
- Taking $\ell = 2$ and $r = 3$, the square-error minimizer of $[3,5]$ is 4.
- Taking $\ell = 3$ and $r = 4$, the square-error minimizer of $[5,7]$ is 6.
- Taking $\ell = 1$ and $r = 3$, the square-error minimizer of $[2,3,5]$ is $10/3$.
- Taking $\ell = 2$ and $r = 4$, the square-error minimizer of $[3,5,7]$ is 5.
- Taking $\ell = 1$ and $r = 4$, the square-error minimizer of $[2,3,5,7]$ is $17/4$.

Thus, $\text{SumSEMs}([2,3,5,7]) = 2+3+5+7+5/2+4+6+10/3+5+17/4 = 505/12$.